Discussion 12 Worksheet

Double integrals

Date: 10/11/2021

MATH 53 Multivariable Calculus

1 Double Integrals

Use geometric arguments to find the values of the following integrals.

1. $\iint_{[0,a]\times[0,b]} c dA$ where a, b, c are all real positive constants.
2. $\iint_{x^2+y^2\leq 1} \sqrt{1-x^2-y^2} dA$
3. $\iint_{x^2+y^2\leq 1} (1-\sqrt{x^2+y^2}) dA$
4. $\iint_{|x|+|y|\leq 1} (1-|x|-|y|) dA$

2 Changing the order of integration

Change the order of integration for these integrals. Sketching the region of integration might be helpful.

(a) $\displaystyle\int_0^1 \int_0^y f(x,y)\, dx\, dy$

(b) $\displaystyle\int_0^{\pi/2} \int_0^{\cos x} f(x,y)\, dy\, dx$

(c) $\displaystyle\int_1^2 \int_0^{\ln x} f(x,y)\, dy\, dx$

3 Double integral practice

Compute these integrals:

(a) $\displaystyle\int_0^1 \int_0^v \sqrt{1-v^2}\, du\, dv$

(b) $\displaystyle\iint_D dA$ where $D = \{(x,y) \mid x^2+y^2 \leq 1\}$ (You can know the answer before doing the computation.)

(c) $\displaystyle\iint_D x\, dA$ where $D = \{(x,y) \mid 0 \leq x \leq \pi, 0 \leq y \leq \sin x\}$

(d) $\displaystyle\iint_D (x+y)\, dA$ where D is bounded by $y = \sqrt{x}$ and $y = x^2$

(e) $\displaystyle\int_0^1 \int_{4y}^4 e^{x^2}\,dx\,dy$

(f) $\displaystyle\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x\,\sqrt{1+\cos^2 x}\,dx\,dy$

4 Challenge

Compute

$$I = \iint_D \sqrt{1-x^2-y^2}\,dA$$

where D is the unit circle without using polar coordinates or geometric arguments. What is the solid whose volume we are computing here?

5 True/False

Supply convincing reasoning for your answer.

(a) T F If $f : \mathbb{R}^n \to \mathbb{R}$ is continuous, then f is the derivative of $\iint f dA$.

(b) T F In some simple cases, computing double integrals reduces to computing the volumes of well-known solids.

(c) T F $\displaystyle\int_0^1 \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dy\,dx = \int_0^1 \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dx\,dy$ by Fubini's theorem.

(Hint: $\frac{d}{dy}\frac{y}{x^2+y^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$*)*

(Hint 2: I wouldn't be giving the above hint if you didn't have to compute the integrals...)

Note: These problems are taken from the worksheets for Math 53 in the Spring of 2021 with Prof. Stankova.